Math1552-Week1

Monday, May 17, 2021 10:23 AM

Math155...
Week1

Welcome to Math 1552

Math 1552 lecture slides adapted from the course materials
By Klara Grodzinsky (GA Tech, *School of Mathematics*, Summer 2021)

NETIQUETTE

Netiquette is the etiquette of online behavior.

In all means of communication in this online course, you will need to follow the same rules of behavior as you would in a face-to-face course when communicating with the other students, teaching assistants, and instructors in the class.

This means that you will have to respect others: negative personal comments are strictly prohibited.

Please also respect your fellow classmates by turning off your microphone and web cam when appropriate.

How will lectures operate?

- Lectures will run "live" at the regularly schedule times.
- Pre-recorded lecture material should be watched prior to the class time (beginning second week).
- Class time will focus on problem solving and answering questions.
- Classes will be recorded if you are unable to attend the live sessions.
- Sessions will run on BlueJeans.

What are studios?

- Studios are problem-based and will run at the scheduled times.
- Your TA will have you work together on weekly problem sets.
- Portions of the studio sessions will be recorded.
- Attendance is required, beginning on the second week of classes.

Syllabus Basics

- Your grade will be determined by:
  - **Participation**
    - Regular homework on MyMathLab (check syllabus schedule)

- Attendance will be taken in studio beginning on ~~May 18~~ second week
- Makeup options for missing class are attending office hours or holding a study session with your classmates
- Lecture Videos
- Lecture Attendance Checks
  - We may require students to complete an in-class poll on Canvas during lecture for credit towards the participation grade (aperiodically scheduled, not announced beforehand)
- **Quizzes and Midterm Exam**
  - Five quizzes (lowest raw score dropped)
  - Midterm exam
- **Final Examination**

Grading Rubric

Assessment	*Weight*
Lecture Participation	5%
Studio Attendance	5%
Studio Participation	10%
Homework	10%
Quizzes Midterm Exam Final Exam	20% 20% 30%

Participation Grade

Homework assignments

Studio: 2X weekly possible for attending and participating in each class, ~~beginning May 18~~) second week

Makeup points can be earned by attending office hours or holding a review session with your classmates

Live Proctoring

- Quizzes and exams will be proctored in a synchronous online session using Honorlock.
- Students must have a broadband internet connection
- Students must have a webcam and microphone
- Students must have a secure private location to take an exam without others in the room
- Students will be asked to provide a picture ID and take a picture of themselves via a webcam as part of the exam process
- Honorlock is not compatible with Linux OS, Virtual Machines, tablets, or smartphones
- Honorlock requires the installation of Google Chrome and the Honorlock Chrome extension.
- Please contact your instructor as soon as possible if you need special accommodations during proctored exams.

Important Websites

- **Course Information:** canvas.gatech.edu
- **Textbook/Homework Access:** http://www.mymathlab.com
- BlueJeans meetings will be used for synchronous class and studio sessions.

Textbook: What to purchase?

- MyMathLab code is a minimum.
  - You may sign up for temporary access if you are not yet sure whether or not you want a printed copy of the text.
  - The bookstore sells codes packaged with a text (cheaper than separate purchases).
  - DO NOT BUY CODES FROM THIRD-PARTY VENDORS! Only online or at the bookstore.

- IMPORTANT: Please register through CANVAS, not the MyMathLab site.

Important Policies

- Make-ups
  - Notify your instructor in advance, if possible
  - Availability determined on a case-by-case basis
  - Might be scheduled at a common time

- Attendance
  - Required in studio and Strongly recommended in lectures

- Health Concerns
  - http://health.gatech.edu/coronavirus/students

- Calculators
  - Not allowed on assessments

Policies (cont.)

- Academic Misconduct

- Any cases will be submitted to the Dean's office.

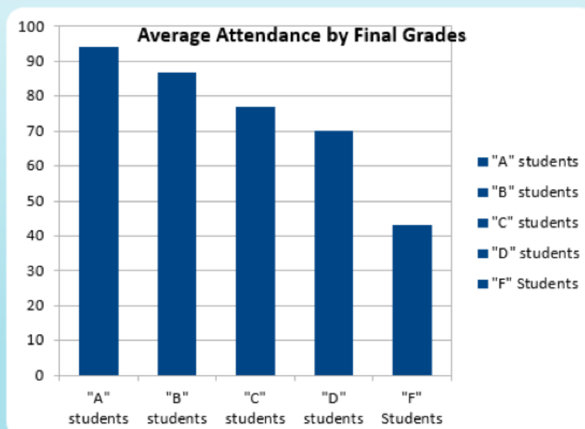
- Disability Services
  - Please discuss any accommodations with me.

- Regrades
  - Submit within one week after receiving your graded paper.

Attendance Matters!

Data from Mrs. G's classes for the past 10 years

Final Grade	Average Attendance
"A" students	94
"B" students	87
"C" students	77
"D" students	70
"F" Students	43



What are the keys to success?

Come to class **and participate!**

Do your homework

Really try to solve the problems, don't just look up the answers

Ask questions

Come to office hours

Go to the math lab virtual hours

Where can I go for extra help?

The instructors and TAs will be holding their scheduled office hours on MS Teams or on BlueJeans.

You may attend office hours for any of the instructors or TAs.

Please reach out to us with any individual needs over email.

What is Integral Calculus?

Math 1552 lecture slides adapted from the course materials
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Course Goals

Topic 1: Integration

- Explore meaning of the integral
  - Finding the area under the curve
  - Using integrals to find volumes
  - Relating the integral to the derivative
- Learn methods of integration
  - Evaluate integrals using various techniques

Course Goals

Topic 2: Infinite Series

- Understand the ideas of convergence and divergence
  - Apply to integrals, sequences, and series
- Use Taylor polynomials to approximate values of functions that cannot be otherwise evaluated by hand

(Taylor polynomials occur in almost every field of study; this is the most important topic in our course!)

Core concept of our class:

Definition: We say the function F is an ***antiderivative*** of the function f if $F'(x)=f(x)$. $(*)$

The collection of all antiderivatives is defined as the indefinite integral of the function:

$\int_a^b f(x)dx$

$$\int f(x)dx = F(x) + C$$

A first example (much more on this later)

$$\frac{d}{dx}\left[x^3 - 2x + 3\right] \xrightarrow{\text{What is?}} G(x) = \int \left(3x^3 - 2\right) dx$$

$\to 3x^2 - 2$

$G(x) = x^3 - 2x + C$

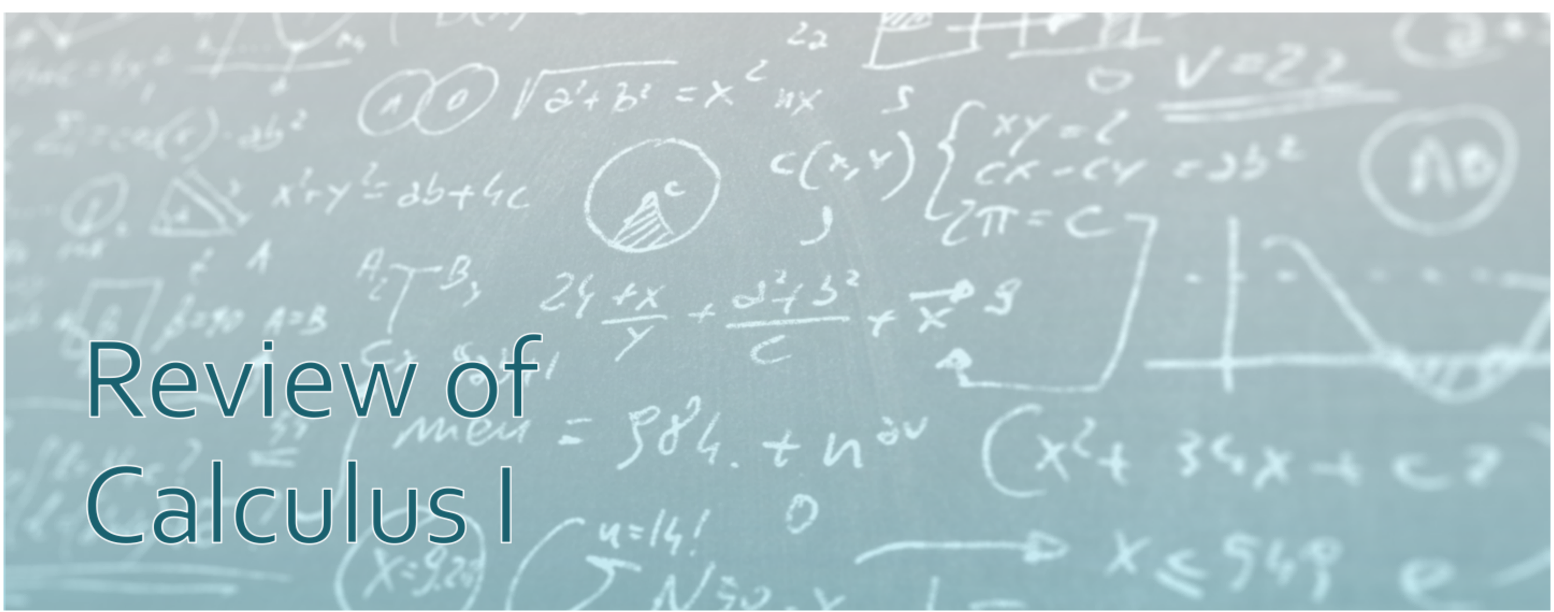
(Why the $+C$?)

"defined equal to"

What is the formula when $G(0) := 3$?

$x=0: \; G(0) = C = 3$

$\to G(x) = x^3 - 2x + 3$



Review of Calculus I

Review these slides on your own!

Derivative Rules (cont.)

Power Rule: $$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

Product Rule:

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

OR

$$\frac{d}{dx}\left[\text{first} \times \text{second}\right]$$

$$= \left(\left[\frac{d}{dx}(\text{first})\right] \times \text{second}\right) + \left(\text{first} \times \left[\frac{d}{dx}(\text{second})\right]\right)$$

Derivative Rules

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

OR

$$\frac{d}{dx}\left[\frac{hi}{lo}\right] = \frac{lo \cdot d(hi) - hi \cdot d(lo)}{lo \cdot lo}$$

The Chain Rule

Let $y = f(u)$ and $u = g(x)$.

Then :

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left[f(g(x))\right] = \left(f'(g(x))\right)\left(g'(x)\right)$$

OR

$$\frac{d}{dx}\left[f(stuff)\right] = f'(stuff) \cdot (stuff)'$$

Some Derivative Formulas (memorize)

$$\frac{d}{dx}\left[e^x\right] = e^x$$ (important formula to know)

$$\frac{d}{dx}\left[\sin x\right] = \cos x \qquad \frac{d}{dx}\left[\sec x\right] = \sec x \tan x$$

$$\frac{d}{dx}\left[\cos x\right] = -\sin x \qquad \frac{d}{dx}\left[\csc x\right] = -\csc x \cot x$$ (less common, but useful to know)

$$\frac{d}{dx}\left[\tan x\right] = \sec^2 x \qquad \frac{d}{dx}\left[\cot x\right] = -\csc^2 x$$ (less common, but useful to know)

(Suggested methods to help memo-ize these formulas – try flash cards or quickly jot five times a week method)

Derivatives of Inverse Functions

Know (memorize) this formula well:

$$\frac{d}{dx}\left[\ln|u|\right] = \frac{1}{u} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left[\ln|u|\right]=\frac{1}{u}\frac{du}{dx}$$

Understand how to derive these formulas quickly:

$$\frac{d}{dx}\left[a^u\right]=a^u\ln(a)\frac{du}{dx}$$

$$\frac{d}{dx}\left[\log_a u\right]=\frac{1}{u\ln(a)}\frac{du}{dx}$$

<u>Note:</u> ln(x) denotes the *natural logarithm* (log base-e) whereas log(x) is notation from the textbook that denotes a base-10 logarithm.

Derivatives of Inverse Functions (formulas to know)

$$\frac{d}{dx}\left[\sin^{-1}(x)\right]=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\cos^{-1}(x)\right]=-\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}(x)\right]=\frac{1}{1+x^2}$$

<u>Example One</u>:

Differentiate the following function:

$$f_1(x)=x^3\tan(x)$$

Example Two:

Differentiate the following function:

$$f_2(x) = \frac{3\sec(x)}{2 + x\cos(x)}$$

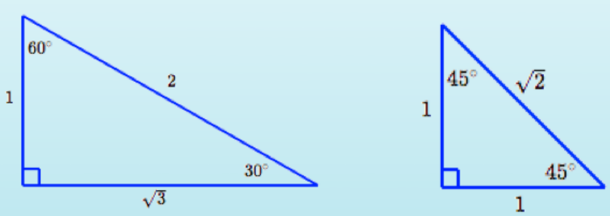
Example Three:

Differentiate the following function:

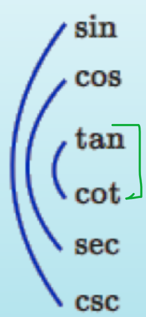
$$f_3(x) = \arctan(\ln(3x+1))$$

Review of Trigonometry

Special right triangles (ratio of sides):



Trig function inverse relationships diagram:



$\cot x = \frac{1}{\tan x}$

Rules to compute trig functions of right triangles:

SOHCAHTOA

Sine **O**pposite **H**ypotenuse **C**osine **A**djacent **H**ypotenuse **T**angent **O**pposite **A**djacent

Credits for figures: https://www.onemathematicalcat.org/Math/Precalculus_obj/trigValuesSpecialAngles.htm

Review of Trigonometry (cont.)

Table of special trig function values (remember them in *radians*):

angle/number	sine $\sin = \frac{\text{OPP}}{\text{HYP}}$	cosine $\cos = \frac{\text{ADJ}}{\text{HYP}}$	tangent $\tan = \frac{\text{OPP}}{\text{ADJ}}$	cotangent (reciprocal of tangent)	secant (reciprocal of cosine)	cosecant (reciprocal of sine)

$0^\circ = 0$ rad	0	1	0	not defined	1	not defined
$30^\circ = \frac{\pi}{6}$ rad	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	2
$45^\circ = \frac{\pi}{4}$ rad	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60^\circ = \frac{\pi}{3}$ rad	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	2	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$90^\circ = \frac{\pi}{2}$ rad	1	0	not defined	0 $(\cot := \frac{\cos}{\sin})$	not defined	1

Credits for table: https://www.onemathematicalcat.org/Math/Precalculus_obj/trigValuesSpecialAngles.htm

Section 4.8: Antiderivatives

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Antiderivatives

Definition: We say the function F is an ***antiderivative*** of the function f if $F'(x)=f(x)$.

The collection of all antiderivatives is defined as the indefinite integral of the function:

$$\int f(x)dx = F(x) + C$$

Some common antiderivatives (memorize)

Function	***Antiderivative***	
$ax^n, n \neq -1$ (n<0)	$a \cdot \frac{x^{n+1}}{n+1}$	(Recall the correct formula when n=1) n=-1? $\int \frac{dx}{x} = \ln\lvert x\rvert + C$
$\sin(x)$	$-\cos(x)$ ✓	
$\cos(x)$	$\sin(x)$ ✓	
$\sec^2(x)$	$\tan(x)$	very often
$\sec(x)\tan(x)$	$\sec(x)$	very often
$\csc^2(x)$	$-\cot(x)$	less common
$\csc(x)\cot(x)$	$-\csc(x)$	less common

Example 1.2:

Find an anti-derivative for the function below.

$$g_1(x) = \sin(x) + \sqrt{x} - \boxed{10e^{-2x}}$$

$$\frac{10e^{-2x}}{-2} + C_3$$

$$\int \sin(x)dx = -\cos(x) + C_1$$

$$\int x^{1/2}dx = \frac{x^{3/2}}{3/2} + C_2$$

$\rightarrow$ In total: $\int g_1(x)dx = -\cos(x) + \frac{2}{3}x^{3/2} + 5e^{-2x} + C$

Example 1.1:

Find an anti-derivative for the function below.

$$g_2(x) = \frac{1}{4x^3} - \sec^2(x)$$

$$= \frac{1}{4}x^{-3} - \sec^2(x)$$

$$\int g_2(x)\,dx = \frac{1}{4}\frac{x^{-2}}{-2} - \tan(x) + C$$

$$= -\frac{1}{8x^2} - \tan(x) + C$$

Example 1.1:

Find an anti-derivative for the function below.

$$g_3(x) = \left(x^2 - \frac{1}{x}\right)^2 \rightarrow \text{"FOIL"}$$

$$= x^4 - \frac{2x^2}{x} + \frac{1}{x^2} = x^4 - \frac{2}{x} + \frac{1}{x^2}$$

$2x^{-1}$ *

x^{-2}

power rule

logarithm case

$$\int g_3(x)\,dx = \frac{x^5}{5} - 2\ln|x| - \frac{1}{x} + C$$

Example 2:

$v(t) = \int a(t)\,dt$ -- velocity

$s(t) = \int v(t)\,dt$ -- position

A particle travels with an acceleration, in meters per second squared, given by:

$$a(t) = t - 5t^2.$$

Find the particle's velocity and position at time t=1 second if the initial position is 2 m and the initial velocity is 10 m/s.

time t=0

$$V(t) = \int (t - 5t^2)\,dt$$
$$= \frac{t^2}{2} - \frac{5t^3}{3} + C_1$$

$$S(t) = \int v(t)\,dt = \frac{t^3}{6} - \frac{5t^4}{12} + C_1 t + C_2$$

IC: $S(0) = \boxed{C_2 = 2}$ ($t=0$)

$V(0) = \boxed{C_1 = 10}$

$\Rightarrow$ ① $S(t) = \frac{t^3}{6} - \frac{5}{12}t^4 + 10t + 2$

② $V(t) = \frac{t^2}{2} - \frac{5}{3}t^3 + 10$

Q: $S(1) = ?$, $V(1) = ?$

<plug-and-chug> :)

Applying the Chain Rule

$$\frac{d}{dx}\left[\cos(ax)\right]$$

$$= \frac{d}{dx}\left[\cos(ax)\right] * \frac{d}{dx}\left[ax\right]$$

$$= -\sin(ax) \cdot a$$

Function	***Antiderivative***
$\sin(ax)$	$-\frac{1}{a}\cos(ax)$
$\cos(ax)$	$\frac{1}{a}\sin(ax)$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)$
$\sec(ax)\tan(ax)$	$\frac{1}{a}\sec(ax)$
$\csc^2(ax)$	$-\frac{1}{a}\cot(ax)$
$\csc(ax)\cot(ax)$	$-\frac{1}{a}\csc(ax)$

More Useful Antiderivative formulas

(~Memorize)

Function	***Antiderivative***
$\boxed{e^{ax}}$	$\boxed{\frac{1}{a}e^{ax}}$
$\frac{1}{x}$	$\ln\|x\|$ ✓
$\frac{1}{\sqrt{1-(ax)^2}}$	$\frac{1}{a}\sin^{-1}(ax)$
$\frac{1}{1+(ax)^2}$	$\frac{1}{a}\tan^{-1}(ax)$
b^{ax}	$\frac{1}{a\ln b}b^{ax}, b>0, b\neq 1$ (Understand how to quickly derive this formula)

In the prev. example: $a=-2$

$\rightarrow \int e^{-2x}\,dx = -\frac{1}{2}e^{-2x}+C$

→ will cover this on Wednesday :)

Example 3.1:

Evaluate the following indefinite integral:

$$\int (2\tan x + 1)\sec^2(x)dx$$

Example 3.2:

Evaluate the following indefinite integral:

$$\int \frac{dx}{\sqrt{16 - x^2}}$$

Example 3.3:

How would you find a formula for the following indefinite integral?

$$\int \frac{dx}{x^2 - x + 1}$$

Section 5.1-5.3: Area under the curve and the definite integral

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Learning Goals

- Understand how to partition an interval
- Draw a picture to approximate the area under the curve with a given number of rectangles
- Compute the Upper and Lower sums
- Calculate the midpoint estimate

Basic Methodology

- *Idea*: Find the area bounded by a function $f(x)$, the lines $x=a$, $x=b$, and the x-axis.

Riemann Sums

- *Idea*: Find the area bounded by a function $f(x)$, the lines $x=a$, $x=b$, and the x-axis.
- *Procedure*: Break the interval $[a,b]$ into n subintervals, and draw a rectangle in each subinterval.
- Summing the areas of the rectangles will approximate the area under the curve.

Defining Sigma Notation

We denote the next (finite) sum of terms by:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

(Practice some sums – examples)

Riemann Sums (cont.)

Upper estimate: use rectangles that over-approximate the area

$$\text{Let } M_i = \max\{f(x)\} \text{ on } [x_{i-1}, x_i].$$

$$\text{Then}: U_f = \sum_{i=1}^{n} M_i \Delta x.$$

Riemann Sums (cont.)

Lower estimate: use rectangles that under-approximate the area

$$\text{Let } m_i = \min\{f(x)\} \text{ on } [x_{i-1}, x_i].$$

$$\text{Then}: L_f = \sum_{i=1}^{n} m_i \Delta x.$$

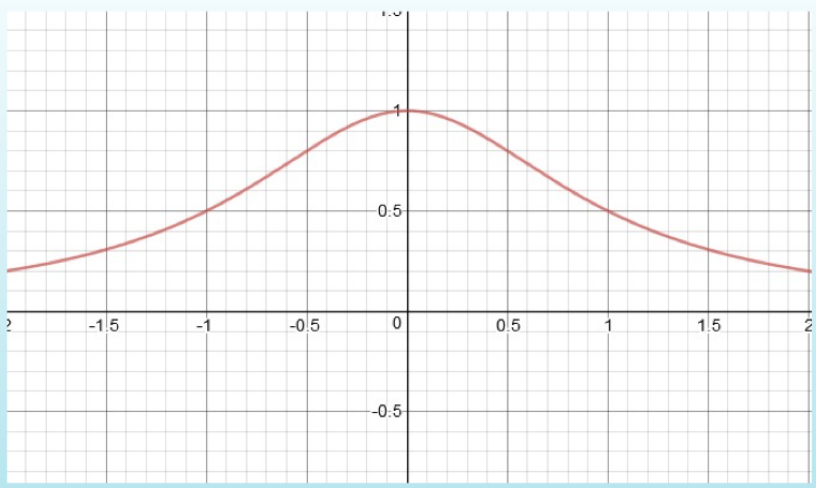
Example 1:

Find the upper and lower sums for the function

$$f(x) = \frac{1}{x^2 + 1}$$

on the interval [-1,2] with n=6 subintervals.

$$f(x)=\frac{1}{x^2+1}$$



As we take rectangles of smaller and smaller width, and then add in more of them to fill up the interval evenly, we get close to the area under the smooth curve.

Key idea: We will compute a definite integral by writing down a Riemann sum for the area approximation and then take a limit as the size width of the rectangles tends to zero.

Midpoint Estimate

Idea: Go for the middle ground approximation (value in between). Plug in the midpoint of each subinterval.

$$\text{On the subinterval } [x_{i-1}, x_i],$$

$$\text{the midpoint is: } \frac{x_{i-1}+x_i}{2}$$

$$\text{and the midpoint sum is:}$$

$$M_n = \sum^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)\Delta x$$

$$M_f = \sum_{i=1} f\left(\frac{\quad}{2} \right) \Delta x$$

Example 2:

Find a midpoint estimate to the area from Example 1.

Recall: We want to approximate the area underneath the function

$$f(x) = \frac{1}{x^2+1}$$

on the interval [-1,2] with n=6 subintervals.

Average Value

The ***average value*** of *f* on [*a*,*b*] is the *y*-value that would generate a rectangle with the same area as *f* on [*a*,*b*].

$$\boxed{AV = \frac{Area}{b-a}}$$

Later:

$$\boxed{AV = \frac{1}{b-a}\int_a^b f(t)dt}$$

Example 3:

Using the same example, find the average value of the function on the interval [-1,2] using the midpoint estimate.

Next Learning Goals

- Be able to find the equation for a general Riemann Sum
- Take the limit of your answer to find the actual area beneath the curve
- Understand the definition of the definite integral
- Understand key properties of the definite integral

General Riemann Sum

Partition the interval $[a,b]$ into n equal pieces:

$a = x_0 < x_1 < x_2 < .... < a_n = b$

Let x_i^* be an arbitrary point in the interval $[x_{i-1}, x_i]$.

Then we can estimate the area under the curve between $x = a$ and $x = b$ with the formula:

$$A \approx \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

Note that: $L_f \le A \le U_f$

What is x_i* in the formula?

A. The left-hand endpoint of the subinterval.
B. The right-hand endpoint of the subinterval.
C. The midpoint of the subinterval.
D. Any value on the subinterval.

The Definite Integral

We define the definite integral to be the limit of the Riemann Sum:

$$\boxed{\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x}$$

The Definite Integral and Area

If the function is always non-negative on [*a*,*b*], we have found ***TOTAL AREA*** under the curve.

$$\int_a^b |f(x)|dx$$

If the function takes on negative values, then we have found the ***NET AREA*** under the curve.

$$\int_a^b f(x)dx$$

Helpful Summation Formulas (memorize)

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$ (Linearity – use to simplify sums)

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$ (Linearity – use to simplify sums)

Example 4:

Use the method of Riemann Sums to evaluate the following definite integral. Choose x_i* to be the left-hand endpoint of each subinterval.

$$\int_{-1}^{2} (x+1)^2\, dx$$

Example 5:

In a memory experiment, the rate of memorization is measured by the function:

$$f(t) = -c\,t^2 + d\,t,\ c>0,\ d>0,$$

where t is the time in minutes, and $f(t)$ is the number of words per minute.

(a) How many words are memorized in the first 2 minutes (from $t=0$ to $t=2$)? USE RIEMANN SUMS.

(b) What is the *average* number of words memorized each minute?

Properties of the Definite Integral

Let $f(x)$ be continuous on $[a,b]$.

$$(1)\int_a^b c\,dx = c(b-a)$$

$$(2)\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$(3)\int_a^a f(x)dx = 0$$

$$(4)\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \text{where } c \in [a,b]$$

Properties of the Definite Integral (cont.)

$$(5)\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$(6)\int_a^b \left[f(x) \pm g(x)\right]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Some More Integral Properties

$$(1)\ \text{If } f(x) \geq 0, \text{ then } \int_a^b f(x)dx \geq 0.$$

$$(2)\ \text{If } f(x) \geq g(x) \text{ on } [a,b], \text{ then}$$

$$\int_a^b f(x)dx \geq \int_a^b g(x)dx.$$

$$(3) \left| \int_a^b f(x)dx \right| \leq \int_a^b |f(x)|dx$$

More Integral Properties (cont.)

$$(4)\ \text{If } f \text{ is an odd function, then}$$

$$\int_{-a}^{a} f(x)dx = 0.$$

$$(5)\ \text{If } f \text{ is an even function, then}$$

$$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx.$$

Example 6:

Given that $\int_1^3 2f(x)dx = 4$ and $\int_1^0 f(x)dx = -1$,

find $\int_0^3 f(x)dx$.

A. -3
B. 1
C. 3
D. 5

Challenge Problem:

Hints:

(1) What are some ways we have seen to simplify this integral?

(2) Recall your special triangle values of trig functions.

$$\int_{-\sqrt{3}}^{\sqrt{3}} \frac{(x^5 + 10x^3 + x^2 + 3x + 1)}{(x^2+1)^2} dx = ?$$

Section 5.4: The Fundamental Theorem of Calculus

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Today’s Learning Goals

- Know the statements of the FTC and the Second FTC
- Apply the FTC to evaluating definite integrals

using the formulas from Section 4.8

- Apply the Second FTC to differentiate an integral

Theorem: The 2nd FTC

Let f be a continuous function on the interval $[a,b]$.

Then if

$$F(x)=\int_a^x f(t)\,dt,$$

$F'(x)=f(x)$ for all x in (a,b).

$$\boxed{\text{i.e., } \frac{d}{dx}\left[\int_a^x f(t)\,dt\right]=f(x).}$$

Example 1: Find F'(2).

$$F(x)=\int_1^x \frac{t}{t^3+3}\,dt$$

A. 2/7
B. 2/11
C. ¼
D. 3/44

Antiderivatives

Definition: We say the function F is an ***antiderivative*** of the function f if $F'(x)=f(x)$.

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From the second FTC, if

$$F(x)=\int_a^x f(t)dt,$$

then F is an antiderivative of f.

The FTC

The Fundamental Theorem of Calculus:

Let f be a function that is continuous on the interval $[a,b]$, and let F be any antiderivative of f. Then:

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a).$$

Example 2: Evaluate.

$$\int_1^3 \frac{1}{x^2}\,dx$$

A. 2/3
B. 4/3
C. 26/9
D. 26/81

Example 3:

The percent of toxin in a lake, where time is in years, is given by the function:

$$f(t) = 50\left(\frac{1}{4}\right)^t.$$

Find the average amount of toxin in the lake between years 1 and 3.

Example 4: Extension to 2nd FTC (chain rule)

Use this extension :

(Does every one see where this comes from?)

$$\frac{d}{dx}\left[\int_{a(x)}^{b(x)} f(t)\,dt\right] = f(b(x))\cdot b'(x) - f(a(x))\cdot a'(x)$$

to find $F'(x)$ if $F(x) = \displaystyle\int_{3x}^{\cos x} \frac{1}{1+t}\,dt.$

Mean Value Theorem

MVT for Integration (statement):

Let f be continuous on $[a,b]$.

Then there exists a $c \in (a,b)$ such that

$$\int_a^b f(x)dx = f(c)(b-a).$$

Example 5:

Find the average value of the function:

$$f(x) = 1 - x^2, \text{-}1 \le x \le 3.$$

Then find a *c* that satisfies the MVT for integration.